ACCUPLACER Study Guide for MAT 090

Solving Formulas for a Specified Variable

Example 1: **Solve** $\boldsymbol{G = w + 43n}$ **for n**

Example 2: **Solve** $\boldsymbol{A = P + Prt}$ **for P**

1. Get all terms with the variable being solved for on one side of the equation. This may involve removing parentheses by distributing.
2. Factor if necessary so the variable is only written once
3. Solve for the variable dividing both sides by the multiplier of the variable

Example 1:

1. Subtract w: $\boldsymbol{G - w = 43n}$
2. Not necessary: $\boldsymbol{G - w = 43n}$
3. Divide by 43: $\boldsymbol{\frac{G-w}{43} = n}$

Example 2:

1. Already done: $\boldsymbol{A = P + Prt}$
2. Factor out P: $\boldsymbol{A = P(1 + rt)}$
3. Divide by $(1 + rt)$: $\boldsymbol{\frac{A}{1+rt} = P}$

<u>Practice Problems:</u>

1. Solve $A = \frac{1}{2}(b_1 + b_2)$ for b_2
2. Solve $ab = d + cb$ for b
3. Solve $F = \frac{mv}{r^2}$ for v

Answers:

1. $b_2 = 2A - b_1$
2. $b = \frac{d}{a-c}$
3. $v = \frac{Fr^2}{m}$

Exponents

The main properties of exponents are:

1. $a^m a^y = a^{m+y}$ Examples: $x^4 x = x^5$, $2^3 2^2 = 2^5 = 32$
2. $\frac{a^m}{a^y} = a^{m-y}$ Examples: $\frac{y^6}{y^2} = y^4$, $\frac{3^5}{3^2} = 3^3 = 27$
3. $a^0 = 1$ Examples: $x^0 = 1$, $3^0 = 1$
4. $a^{-n} = \frac{1}{a^n}$ Examples: $x^{-2} = \frac{1}{x^2}$, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
5. $(a^n)^m = a^{nm}$ Examples: $(y^2)^5 = y^{10}$, $(2^2)^{-3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$
6. $(ab)^m = a^m b^m$ Examples: $(xy)^5 = x^5 y^5$, $(3 \cdot 2)^2 = 3^2 2^2 = 9 \cdot 4 = 36$

These properties can be combined to simplify expressions.

Example 1: Simplify $\frac{24a^5b^3}{-8a^4b}$

Use property 2 to simplify the a and b terms separately:

$$\frac{24a^5b^3}{-8a^4b} = \frac{24}{-8}a^{5-4}b^{3-1} = -3ab^2$$

Example 2: Simplify $\frac{(x^5)^2(x^{-3})^4}{(x^2)^3}$

1. Use property 5

$$\frac{x^{10}x^{-12}}{x^6}$$

2. Use property 1 in the numerator

$$\frac{x^{10+-12}}{x^6} = \frac{x^{-2}}{x^6}$$

3. Use property 2

$$x^{-2-6} = x^{-8}$$

4. Use property 4 $\quad x^{-8} = \frac{1}{x^8}$

Practice Problems:

Simplify: 1. $\frac{2^7}{2^{-2}}$ 2. $\frac{(5xy)^2}{x^3}$ 3. $\frac{12b}{2a^2b^4}$ 4. $\frac{(2x^2y)^2}{(xy)^3}$

Answers: 1. $2^9 = 512$ 2. $25x^{-1}y^2$ 3. $6a^{-2}b^{-3}$ 4. $4xy^{-1}$

Polynomials: Combining Like Terms

Example: **Simplify** $3x^2 + 4x - 5 + 3(x-3) - 2(x^2 + 4x - 1)$

1. Get rid of any parentheses by distributing
2. Group like terms

3. Combine like terms by adding/subtracting their coefficients

Example:

1. $\mathbf{3x^2 + 4x - 5 + 3x - 9 - 2x^2 - 8x + 2}$
2. $\mathbf{(3x^2 - 2x^2) + (4x + 3x - 8x) + (-5 - 9 + 2)}$
3. $\mathbf{x^2 - x - 12}$

<u>Practice Problems:</u>

Simplify:

1. $5a + 6 - 4 + 2a^3 - 6a + 2$
2. $(a^2 - 3b^2 + 4c^2) - (-5a^2 + b^2 + 2c^2)$
3. $(5x^2 + 7) - (4x + 9) + (x^2 - 3x)$
4. $(2x)^2 + 3x - 4x^2 + 2 - x$

Answers:

1. $2a^3 - a + 4$
2. $6a^2 - 4b^2 + 2c^2$
3. $6x^2 - 7x - 2$
4. $2x + 2$

Polynomials: Multiplying

Example 1: $\mathbf{2x(x^2 + 3x + 4)}$

To multiply a monomial (one term) by a polynomial, simply distribute the monomial through the parentheses and multiply using the rules of exponents.

$$\mathbf{(2x)(x^2) + (2x)(3x) + (2x)(4) = 2x^3 + 6x^2 + 8x}$$

Example 2: $\mathbf{(2x + 3)(4x - 1)}$

When multiplying two binomials (two terms in each set of parentheses), most people remember the FOIL method. First - Outer - Inner - Last.

First: Multiply the first terms in each binomial. In our example, we have $\mathbf{(2x)(4x) = 8x^2}$

Outer: Multiply the two outermost terms. $\mathbf{(2x)(-1) = -2x}$

Inner: Multiply the two innermost terms. $\mathbf{(3)(4x) = 12x}$

Last: Multiply the last terms in each binomial. $\mathbf{(3)(-1) = -3}$

Add all of the results and combine like terms. $\mathbf{8x^2 - 2x + 12x - 3 = 8x^2 + 10x - 3}$

Example 3: $(\mathbf{x+2})(\mathbf{2x^2+5x-1})$

The FOIL method only works when we have 2 binomials. Here, we're multiplying a binomial and a trinomial. In this case, we want to split up and distribute the x and the 2 from the binomial.

$\mathbf{x(2x^2+5x-1)+2(2x^2+5x-1)}$

Then we just distribute and combine like terms

$\mathbf{2x^3+5x^2-x+4x^2+10x-2}$

$\mathbf{2x^3+9x^2+9x-2}$

<u>Practice Problems:</u>

1. $(x+3)(x+5)$ 2. $(a+3)(a^2-4a+2)$ 3. $(a-b)(a^2+ab+b^2)$

Answers:

1. $x^2+8x+15$ 2. $a^3-a^2-10a+6$ 3. a^3-b^3

Polynomials: Factoring

Factoring is the reverse of multiplication.

Factoring out a common factor

Example: Factor out a common factor: $5x^3-10x^2+20x$

1. Notice that there is a common factor in each term and factor it out of each term
2. Use the distributive law to rewrite the expression

1. There is a 5x in each term. We can rewrite the expression: $5x(x^2)-5x(2x)+5x(4)$
2. Use the distributive law: $5x(x^2-2x+4)$

Factoring a trinomial (reverse FOIL): $\boldsymbol{ax^2+bx+c} \rightarrow (?+?)(?+?)$

Example: Factor $3x^2+10x-8$

1. Factor out a common factor if possible
2. Find two numbers that multiply to ax^2 term and put them in the first slot of each binomial
3. Find two numbers that multiply to c and put them in the last slot of each binomial.
4. FOIL your guess to see if you get the correct middle term. If so, you're done! If not, try another pair of numbers.

Example: $3x^2+10x-8$

1. Not possible
2. $3x$ and x are the only good terms that multiply to $3x^2$: $(3x+?)(x+?)$
3. There are several possibilities for 2 numbers that multiply to -8:
   $(1)(-8), (-1)(8), (8)(-1), (-8)(1), (2)(-4), (-2)(4), (4)(-2), (-4)(2)$
   We need the pair that will FOIL out to give us $3x^2+10x-8$.

4. Try $(1)(-8)$: $(3x+1)(x-8)=3x^2-24x+x-8$… This has the wrong middle term
   Try $(2)(-4)$: $(3x+2)(x-4)=3x^2-12x+2x-8=3x^2-10x-8$… close, off by a $(-)$
   Try $(-2)(4)$: $(3x-2)(x+4)=3x^2+12x-2x-8=3x^2+10x-8$… HOORAY!

   Answer: $3x^2+10x-8=(3x-2)(x+4)$

Please note that many "shortcuts" to this process exist. This guess and check method helps you understand the process while in time allowing you to factor very quickly… it just takes practice!

Perfect Square Trinomials

Example: Factor x^2+6x+9

By using the method above, you will find that this can be factored as follows:

$$x^2+6x+9=(x+3)(x+3)$$

Since the two binomial factors are the same, the answer can be written

$$(x+3)^2$$

When a trinomial can be factored in this way, it is called a perfect square trinomial.

Difference of Squares

Example: Factor $x^2 - 16$.

When a polynomial has two terms that are perfect squares that are subtracted from each other, we have the special case of the difference of squares. In general, a difference of squares can be written

$$A^2 - B^2$$

A difference of squares can be factored:

$$A^2 - B^2 = (A + B)(A - B)$$

Example: Notice that x^2 and 16 are both perfect squares and that they are subtracted. Thus $x^2 - 16$ is a difference of squares and can be factored:

$$x^2 - 16 = (x + 4)(x - 4)$$

Practice Problems:

Factor:

1.	$x^3 - 2x^2 + x$	2.	$3x^3 - 6x^2$	3.	$3a^2 - 10a + 8$
4.	$y^7 + 5y^6 - 14y^5$	5.	$4x^2 - 36$	6.	$w^2 - 14w + 49$

Answers:

1.	$x(x-1)^2$	2.	$3x^2(x-2)$	3.	$(3a-4)(a-2)$
4.	$y^5(y+7)(y-2)$	5.	$4(x+3)(x-3)$	6.	$(w-7)^2$

Solving Quadratic Equations

Example 1: Solve for x. $\mathbf{3x^2 - 6x - 1 = 0}$

Example 2: Solve for x. $\mathbf{x^2 - 3x = 6}$

1. Get the equation in standard form - $ax^2 + bx + c = 0$. Do this by expanding and moving all terms to one side of the equation if necessary.
2. When solving the quadratic equation, $ax^2 + bx + c = 0$, you use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1:

1. Already in standard form: $\mathbf{3x^2 - 6x - 1 = 0}$
2. Substitute $\mathbf{a = 3, b = -6, c = -1}$ into the quadratic formula

$$\mathbf{x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}}$$

$$\mathbf{x = \frac{6 \pm \sqrt{48}}{6} = 1 \pm \frac{\sqrt{48}}{6} = 1 \pm \frac{\sqrt{(16)(3)}}{6} = 1 \pm \frac{2\sqrt{3}}{3}}$$

$$\mathbf{x = 1 + \frac{2\sqrt{3}}{3} \quad and \quad x = 1 - \frac{2\sqrt{3}}{3}}$$

Example 2:

1. Subtract 6: $\mathbf{x^2 - 3x - 6 = 0}$
2. Substitute $\mathbf{a = 1, b = -3, c = -6}$ into the quadratic formula

$$\mathbf{x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-6)}}{2(1)}}$$

$$\mathbf{x = \frac{3 \pm \sqrt{33}}{2}}$$

$$\mathbf{x = \frac{3+\sqrt{33}}{2} \quad and \quad x = \frac{3-\sqrt{33}}{2}}$$

<u>Practice Problems:</u>

Solve for the given variable.

1. $x^2 + 7x - 3 = 0$ 2. $3y^2 = 18y - 6$ 3. $7x(x+2) + 5 = 3x(x+1)$

Answers:

1. $x = -\frac{7}{2} + \frac{\sqrt{61}}{2}$ and $x = -\frac{7}{2} - \frac{\sqrt{61}}{2}$

2. $y = 3 + \sqrt{7}$ and $y = 3 - \sqrt{7}$

3. $x = -\frac{11}{8} + \frac{\sqrt{41}}{8}$ and $x = -\frac{11}{8} - \frac{\sqrt{41}}{8}$

Lines: Slope, Intercept, and Graphing

Any equation that can be written in the form $Ax + By = C$ is a line in the x-y plane. But we usually see the equation for a line in a different form:

<u>Slope-Intercept Form</u>: $y = mx + b$ where m is the slope of the line and b is the y-intercept

<u>Slope:</u>

There are many ways to think about slope: $m = \frac{rise}{run} = \frac{change\ \ in\ y}{change\ \ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Conceptually, slope is a measure of the "steepness" of a line.

<u>Intercepts:</u>

The x-intercept is the point where a graph crosses the x-axis. You can find it by setting x=0 and solving for y.

The y-intercept is the point where a graph crosses the y-axis. You can find it by setting y=0 and solving for x.

Example 1: Find the slope between the points $(1, -3)$ and $(-3,5)$.

For this problem, I will call $(x_{1,}y_1) = (1, -3)$ and $(x_{2,}y_2) = (-3,5)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - -3}{-3 - 1} = \frac{8}{-4} = -2$$

Example 2: Find the y-intercept of the line between the points $(1, -3)$ and $(-3,5)$.

We already found that the slope between the points is -2. We can plug this into our slope-intercept equation to get:

$$y = -2x + b$$

All we need to do now is find b. To do this plug either point $(1, -3)$ or $(-3,5)$ in for x and y, and solve for b:

$$-3 = -2(1) + b \rightarrow b = -1$$

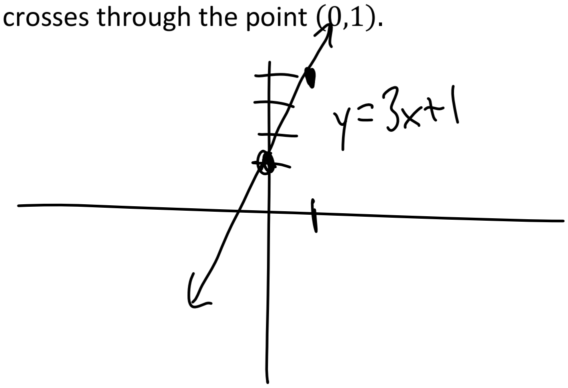
Example 3: Find the equation of the line between the points $(1, -3)$ and $(-3,5)$.

We found m and b in the previous 2 examples! Now just plug m and b into the slope intercept equation and you're done!

$$y = -2x - 1$$

Example 4: Graph the line $y = 3x + 1$.

Since the equation is in slope intercept form, we know that the slope is 3 and its y-intercept is 1. Since the slope is 3, the line increases by 3 units for every 1 unit it moves to the right. Since the y-intercept is 1, we know the graph crosses through the point $(0,1)$.

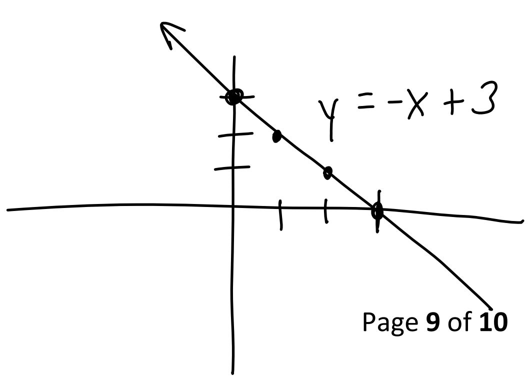


<u>Practice Problems:</u>

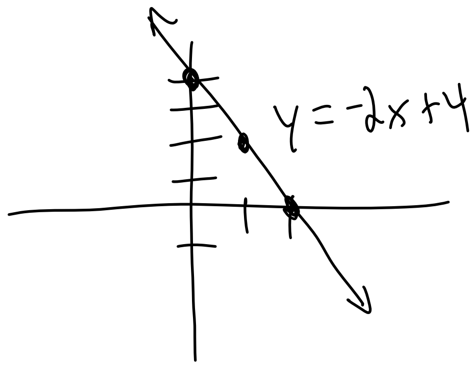
1. Find the slope and y-intercept of the line $y = -x + 3$. Graph it.
2. Find the slope and y-intercept of the line $4x + 2y = 8$. Graph it.
3. Find the slope of the line between the points $(-2,1)$ and $(3,5)$.
4. Find the equation of the line between the points $(-2,1)$ and $(3,5)$.

Answers:

1. $m = 1, b = 3$



2. $y = -2x + 4$ so $m = -2, b = 4$



3. $m = \frac{4}{5}$

4. $y = \frac{4}{5}x + \frac{13}{5}$