Room For Two:

Standing On the Shoulders of Giants

A brief examination of the calculus as it pertains to Leibniz, Newton, and the feud between their followers.

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Terry Reeves, Instructor

Red Rocks Community College

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Paige Cudworth, Instructor

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"As God calculates, so the world was made."

- Gottfried Wilhelm Leibnitz[1]

The story of "the calculus"[2](henceforth referred to as simply: "calculus") is not as sweet as your average bedtime story. This is because an embittered war was waged over the glory of discoveringcalculus. To begin to examine this phenomenon, we must first answer the question… What is calculus? To begin to answer this question we must also determine the originality of the idea of calculus as a mathematical concept.Finally,we will examine the elements that led to the great calculus wars, which were the feud over the priority of the discovery of calculus. The Calculus war pitted the great mathematicians Sir Isaac Newton and Gottfried Wilhelm Leibnitz against one another.

What is Calculus?

What is calculus? Why is it so important? Calculus is the mathematics of change. It can be used to calculate the areas of unusual shapes, the slope of a tangent line to a curve, find the instantaneous rate of change of a function, find optimum values of a function,

[1](Motz and Weaver 125)

[2] What is commonly referred to as calculus, scholars usually distinguish as "the calculus" because any method of calculation can be referred to as calculus. (Kelly)

and so on. Calculus can also to solve real-world problems, such as the breaking power needed to stop a racecar. The list of the uses of calculus continues to grow seemingly infinitely. However, to truly understand the nature of calculus we must first briefly look at the mathematics prior to the discovery of calculus. Though by no means the absolute beginning of humankind's mathematical awareness, for the sake of conciseness let us begin with the Greeks. The Greek mathematicians were the first on record to develop the concept of the infinite series by exploring such paradoxes as Zeno's Dichotomy.

> *Zeno's Dichotomy briefly stated is: if (a) the idea that a number can be divided in half an infinite amount of times,and if (b) distances can be represented numerically as units of measure, consider the following situation:If a wall is 20ft away from where you stand and you walk half the distance to the wall you are now 10ft away from the wall. Again you walk half the distance from where you now stand to the wall and you are now 5ft away from the wall. You repeat the process again and you are now* $2\frac{1}{2}$*ft away from the wall. Keeping in mind statement (a)that a number can be divided in half an infinite amount of times, and statement (b) that distances can be represented numerically, how could you reach the wall to make the distance between you and the wall 0, when the distance to be traveled persist infinitely?*

Zeno's dichotomy and other ideas have inspired work around the topic of infinity. The notion of infinite series was also explored by what became known as "The Method of Exhaustion." For example, to find the area and

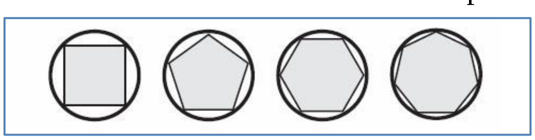

Figure 1: The Method of Exhaustion *Source:*(Kelly)*, Fig.1.5*

volume of a figure such as a circle by inscribing familiar figures, for which the methods for calculating area are known, within the circle such as a rectangle, an approximation for the area of the circle could be estimated; however, there is still some area outside of the rectangle yet inside of the circle that remains unaccounted for. A more accurate estimate can be achieved by continuing to add sides to the inscribed rectangle, to form a hexagon for example, and continue still, adding an increasing number sides to form different polygons, thus minimizing the amount of "unaccounted" space between the interior of the circle and the exterior of the inscribed polygon.As the number of sides of the polygon increases, the more accurate is the approximation of the area of the circle. If you let the amount of sides increase without bound or infinitely (an idea that came after the Greeks), the inherent limit of the areas of the polygon would be the area of the circle.(Stewart 3)An example of the method of exhaustion is illustrated in Figure 1.

Understanding the concept of the method of exhaustion, let us next turn our attention to the "falling apple problem."This problem lead Newton to the discovery of the law of gravity and ultimately to calculus. By observation we can conclude that a falling object, in this case an apple, increases its speed (distance traveled/time) as it falls to the ground. We could take the speed during one second of the fall and be left with an average speed during the one second interval. We could gain a closer approximation to the speed by allowing the intervals that we test to become smaller and smaller. However, if we wanted to know the speed of the object at a precise moment of time, an "instantaneous velocity," we would need to allow our intervals of time to become infinitesimally smallthus, allowing our interval to seemingly go to$\frac{0}{0}$. However, this is not the case. Our interval only approaches $\frac{0}{0}$ but does not actually equal zero. This idea led to using the slope of a tangent line to calculate the slope of a point on a non-linear curve as well.

The Antagonist of the Feud

Enter into our story, Isaac Newton, the young British natural philosopher who would subsequently make major contributions to the fields of math and science. To briefly explain the motivations that led Newton to discover his method of calculus, it should be said that there weretwo major problems of the 17^{th} Century that sparked Newton's interest: The Tangent Line problem and The Motion of Planets. Essentially, Newton was primarily motivated by rates of change. Newton's terminology to describe his calculus is based on his obsession with time: Denoting time as "flux"(Schaffer, Fara and Stedall) in his work and what are essentially the equivalent to today's derivatives he called "fluxions." Thus, Newton's concept of what would become known as calculus was based on rates of change in time. Newton would be akin to the applied mathematicians today, in terms of his methodology, because he applied calculus to real world problems.

Alternatively, we have Gottfried Wilhelm Leibnitz, a German philosopher of law, religion, politics and the natural sciences of which he was primarily self-taught. Leibnitz, who was already an accomplished scholar in other disciplines, often rediscovered the works of others due to him being primarily self-taught in the field of mathematics. Leibnitz also was trying to solve one of the great problems of his time, finding the area of a region under a curve. To solve this problem, Leibnitz developed a method of dividing the region under the curve into recognizable segments, in this case rectangles. Leibnitz called the sum of the areas of these segments "omnia."

Leibnitz understood that if he let the number of segments become infinite, then the resultwould be the exact area of the region under the curve. Likewise, Leibnitz also understood just as the sum of the areas of an infinite numberof infinitely small segments would result in the exact value of the area of the region, the difference of the areas of the two segments represent a rate of change of the curve over an interval. Likewise, when the difference of two partitions that are infinitely close together is taken, a tangent is produced.Leibnitz's analysis of the calculus led to a superior notation system than that of Newton's. The $\frac{dy}{dx}$ notation, and the "omnia" or integral sign $\int$ all came from the mind of Leibnitz. To contrast Newton, Leibnitz would be more akin to a pure mathematician of today, in terms of methodology, because he saw calculus as a means to solve purely mathematical problems, such as areas under curves.

The Claim to Calculus

While it is now believed that both men independently discovered calculus, a great feud between Leibnitz and Newton ensued over the priority of the discovery. Leibnitz published his work on calculus in 1684 and it was spread throughout Europe through his association with the Bernoulli family, a powerful mathematical dynasty. It is said that Newton had discovered the preliminary ideas of calculus some ten years before Leibnitz published his own work, but he was slow to publish his own work for fear of ridicule. It wasn't until Newton published his own description of calculus as an appendix in his book on optics, that the priority over the discovery of calculus had come into question. In this book he added a comment that implied that Leibnitz had plagiarized his work. Thus began a bitter feud between two great mathematical minds that was instigated by their followers and fellow countrymen. The feud went on and on for years until Leibnitz filed a dispute with the Royal Society, a collection of scholars of which both Newton and Leibnitz were members. Newton however was currently serving as the society's president when the dispute came before the Royal Society's review panel. Newton did not let such a meaningless thing as conflict of interest prevent him from participation in the Society's review of the dispute, though. In fact, Newton himself wrote the Society's response and was the only member to sign it.The Society found in favor of Newton. The Royal Society having ruled for Newton tarnished the reputation of Leibnitz, leaving him a bitter man. In the end, Newton's calculus succumbed to Leibnitz's superior notation, which is still in use today; however, when the two men died, Newton was buried in Westminster Abby, with all the accolades of a General, while Leibnitz was buried much more humbly. "At his funeral there was one mourner, his secretary. An eyewitness stated, 'He was buried more like a robber than what he really was—an ornament to his country'" (Anton, Bivens and Davis 6). All in all, none of the contributionsboth Leibnitz and Newton have made would have been possible without the mathematicians that preceded them. In the classic text *What is Mathematics?*, authors Richard Courant and Herbert Robbins state in reference to the "invention" of calculus: "In reality, the calculus is the product of a long evolution that was neither initiated nor terminated by Newton and Leibniz, but in which both played a decisive part." (Courant and Robbins 398)Newton was known to have said to Robert Hooke, "If I have seen farther than Descartes, it is because I have stood on the shoulders of giants."(Boyer and Mezrbach 392) Even today the calculus wars still rage on among some historians, scholars, fans of Leibniz, and fans of Newton. However, time has proven that on top of the shoulders of giants, there is room for two.

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